VEERMATA JIJABAI TECHNOLOGICAL INSTITUTE

Mathematics Department
Assignment –I
Complex Numbers
Batch 2 & 6
Date of Submission: 27/09/2019

1. Prove that: $\left(\dfrac{1+\cos\frac{\pi}{9}+i\sin\frac{\pi}{9}}{1+\cos\frac{\pi}{9}-i\sin\frac{\pi}{9}}\right)^{18}=1$

2. If $a\cos\alpha+b\cos\beta+c\cos\gamma=0,\ a\sin\alpha+b\sin\beta+c\sin\gamma=0$

Prove that $a^3\cos3\alpha+b^3\cos3\beta+c^3\cos3\gamma=3abc\cos(\alpha+\beta+\gamma)$,

$a^3\sin3\alpha+b^3\sin3\beta+c^3\sin3\gamma=3abc\sin(\alpha+\beta+\gamma)$.

3. If $x_r=\cos\frac{\pi}{3^r}+i\sin\frac{\pi}{3^r}$, prove that

(a) $x_1x_2x_{3\ldots\ldots}\infty=i$ (b) $x_0x_1x_2x_{3\ldots\ldots}\infty=-i$

4. By using De Moivre's Theorem, show that $\cos\alpha+\cos2\alpha+\ldots\ldots\cos5\alpha=\dfrac{\cos3\alpha\sin\left(\frac{5\alpha}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)}$.

5. If $\sin6\theta=a\cos^5\theta\sin\theta+b\cos^3\theta\sin^3\theta+c\cos\theta\sin^5\theta$, Find the values of a,b,c.

6. Prove that $\dfrac{1+cos7\theta}{1+cos\theta}=\left(x^3-x^2-2x+1\right)^2$, *where* $x=2\cos\theta$

7. Find all the values of $\sqrt[3]{1+i\sqrt{3}}+\sqrt[3]{1-i\sqrt{3}}$ and find the continued product of these values.

8. If $\alpha,\alpha^2,\alpha^3,\alpha^4$ are the roots of $x^5-1=0$, find them and prove that

$(2-\alpha)(2-\alpha^2)\ldots\ldots(2-\alpha^4)=31.$

9. Solve the equation: $x^5+x^4+x^3+x^2+x+1=0.$

10. Prove that $\cos^6\theta-\sin^6\theta=\frac{1}{16}(\cos6\theta+15\cos2\theta)$.

11. Prove that $\sin^7\theta\cos^3\theta=-\frac{1}{512}(\sin10\theta-4\sin8\theta+3\sin6\theta+8\sin4\theta-14\sin2\theta)$.

12. Prove that $\cos ec\, h\, x + \cot h\, x = \cot h \frac{x}{2}$.

13. Prove that $\sinh^5 x = \frac{1}{16}\left[\sinh 5x - 5\sinh 3x + 10\sinh x\right]$.

14. If $\cosh u = \sec\theta$, then prove that $u = \log\left[\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right]$.

15. If $\log\tan x = y$, prove that $\cosh ny = \frac{1}{2}\left[\tan^n x + \cot^n x\right]$ and

$\sinh(n+1)y + \sinh(n-1)y = 2\sinh ny \cos ec 2x$

16. Prove that $16\sinh^5 x = \sinh 5x - 5\sinh 3x + 10\sinh x$

17. If $\tanh(\alpha + i\beta) = x + iy$, prove that $x^2 + y^2 + 1 = 2x\coth 2\alpha$ & $x^2 + y^2 + 2y\cot 2\beta = 1$.

18. Separate into real and imaginary parts $\sin^{-1} e^{i\theta}$.

19. Separate into real and imaginary parts $\tan^{-1} e^{i\theta}$.

20. Separate into real and imaginary parts of $\sec(x+iy)$.

21. If $\cos(\alpha + i\beta) = x + iy$, prove that $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$, $\frac{x^2}{\cosh^2\beta} + \frac{y^2}{\sinh^2\beta} = 1$.

22. If $\sin^{-1}(\alpha + i\beta) = x + iy$, show that $\sin^2 x$ & $\cosh^2 y$ are the roots of the equation

$\lambda^2 - \left(\alpha^2 + \beta^2 + 1\right)\lambda + \alpha^2 = 0$.

23. Separate into real and imaginary part $\log_{(1-i)}(1+i)$.

24. Prove that the principal value of $(1 + i\tan\alpha)^{-i}$ is $e^{\alpha}[\cos(\log\cos\alpha) + i\sin(\log\cos\alpha)]$

25. If $x = 2\cos\alpha\cosh\beta$, $y = 2\sin\alpha\sinh\beta$, prove that $\sec(\alpha + i\beta) + \sec(\alpha - i\beta) = \frac{4x}{x^2 + y^2}$.

26. Determine all integer values of θ with $0 \le \theta \le 90$ for which $\left(\cos\theta + i\sin\theta\right)^{75}$ is a real number.

27. The equation $z^{10} + (13z-1)^{10} = 0$ has ten roots $z_1, \overline{z}_1, z_2, \overline{z}_2, ..z_2, \overline{z}_5$. Find the value of

$\frac{1}{z_1\overline{z}_1} + \frac{1}{z_2\overline{z}_2} + \frac{1}{z_3\overline{z}_3} + \frac{1}{z_4\overline{z}_4} + \frac{1}{z_5\overline{z}_5}$

28. Find the roots of the equation $z^6 + z^4 + z^3 + z^2 + 1 = 0$ that have positive real parts.